The usage of technology to revive classical topics in mathematics

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CADGME 6
Sapientia University
Târgu Mureş/Marosvásárhely, Romania

JERUSALEM COLLEGE OF TECHNOLOGY
LEV ACADEMIC CENTER

« Classical » teaching of mathematics

- Every domain is taught separately (Bourguignon 2002):
  - Linear Algebra
  - Calculus
  - Probability
  - Geometry
  - Etc.

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  - Etc.
- « Applications of Calculus to Geometry » = Elementary Differential Geometry
- Linear Algebra and Geometry: the book by Jean Dieudonne.
- Today?

The usage of software: some remarks from the past

The design principles of the software (Yerushalmy 1999):

- Schwartz 1995: Tools for doing and tools for learning
- B. Leong (plenary address at the 3rd Asian Technology Conference in Mathematics, Tsukuba, Japan, 1998): "It has to be more efficient in terms of performance time, it has to include as much math topics as possible, be a slick and easy to use consumer product and it should not introduce many changes in new versions so that the customer will be able to easily move from version to version".

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MAYBE THE SAME TOOL HAS TWO GOALS???

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BLACK BOX???

Claim

The multi-purpose nature of a CAS enables to build bridges between mathematical domains which are generally taught separately.

The various registers of representation provided by CAS and other kinds of software, and their versatility enabling to switch between them correspond to the required skills for building these bridges.

For this, technology has to be used not as a bypass for a lack of theoretical knowledge, but as a **facilitator** to enhance new mathematical knowledge and more profound conceptual understanding.

Revival: what do we mean?

- To pour new contents into a classical topic
- To have a new insight into a classical topic
- To bring an old-classical topic which has been « forgotten » back to the stage

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Parametric integrals

Differential Geometry:
- **Isoptic curves**
- **Envelopes**

Parametric definite integrals

- Important in Physics, and in general in engineering domains
- Paper-and-pencil work
- Computing using a CAS:
  - Direct computations
  - Usage of an implemented tutorial
    - Hints: which method may be used
    - If multiple methods are proposed, identities (integral, combinatorial) may be derived
  - Find (other) mathematical meanings using other ICTs
    - History of mathematics (MacTutor or else)
    - Dictionary of mathematical notions
    - …
  - **Recall Janos Karsai's remark in his talk this morning @ CADGME 6 about Science courses for math students!!!!**
  - **DP 2004,2005,2009,2011 – DP & Zeitoun 2011,2015,2016**

Tool to do = Tool to learn

Example

$$I_n = \int_0^{\pi/4} \tan^n x \, dx$$

Trials with a CAS

$$paramint := \int_0^{\frac{Pi}{4}} \tan(x)^n \, dx;$$

$$\int_0^{\frac{1}{4}\pi} \tan(x)^n \, dx$$

for k **from** 0 **to** 8 **do** $eval(paramint, n = k)$ **end do**;

$$\frac{1}{4}\pi$$

$$\frac{1}{2}\ln(2)$$

$$1 - \frac{1}{4}\pi$$

$$\frac{1}{2} - \frac{1}{2}\ln(2)$$

$$-\frac{2}{3} + \frac{1}{4}\pi$$

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$$\frac{13}{15} - \frac{1}{4}\pi$$

$$\frac{5}{12} - \frac{1}{2}\ln(2)$$

$$-\frac{76}{105} + \frac{1}{4}\pi$$

We identify the subsequences discovered previously →

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We identify the subsequences discovered previously →

Actually, we could have begun with the CAS discovery, then find « manually » formulas for the sequences

The sequence of denominators

Search: **seq:1,3,15,105,315,3465,45045,45045**

Displaying 1-1 of 1 result found. page 1

Sort: relevance | references | number | modified | created Format: long | short | data

A025547 Least common multiple of {1,3,5,...,2n-1}. +20 27

1, 3, 15, 105, 315, 3465, 45045, 45045, 765765, 14549535, 14549535, 334639305, 1673196525, 5019589575, 145568097675, 4512611027925, 4512611027925, 4512611027925, 166966608033225, 166966608033225, 6845630929362225 (list; graph; refs; listen; history; text; internal format)

OFFSET 1,2

COMMENTS This sequence coincides with the sequence f(n) = denominator of 1+1/3+1/5+1/7+...+1/(2n-1) iff n <= 38. But a(39) = 6414924694381721303722858446525, f(39) = 583174972216520118520259858775. - T. D. Noe, Aug 04 2004

Coincides for n=1..42 with the denominators of a series for Pi*sqrt(2)/4

MAPLE
```
A025547:=proc(n) local i, t1; t1:=1; for i from 1 to n do t1:=lcm(t1,
  2*i-1); od: t1; end;
f := n->denom(add(1/(2*k-1), k=0..n)); # a different sequence!
```

CROSSREFS Cf. A007509, A025550, A075135. The numerators are in A074599.
Cf. A003418 (LCM of {1..n}).
Cf. A005408.

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The sequence of numerators

Search: **seq:1,2,13,76,263,2578**

Displaying 1-1 of 1 result found. page 1

Sort: relevance | references | number | modified | created Format: long | short | data

A007509 Numerator of Sum_{k=0..n} (-1)^k/(2*k+1). +20
(Formerly M2061) 11

1, 2, 13, 76, 263, 2578, 36979, 33976, 622637, 11064338, 11757173, 255865444, 1346255081, 3852854518, 116752370597, 3473755390832, 3610501179557, 3481569435902, 133330680156299, 129049485078524, 5457995496252709 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,2

COMMENTS Denominators of convergents to 4/pi.

MAPLE A007509 := n->numer(add((-1)^k/(2*k+1), k=0..n));

CROSSREFS Denominators are A025547.
Contribution from Johannes W. Meijer, Nov 12 2009: (Start)
Cf. A157142 and A166107.
Appears in A167576, A167577, A167578, A024199, A167588 and A167589.
(End)

The two sequences are strongly connected:

1. We knew that previously
2. Students may have an indication to look for something else

Concrete meaning
an application in soil mechanics

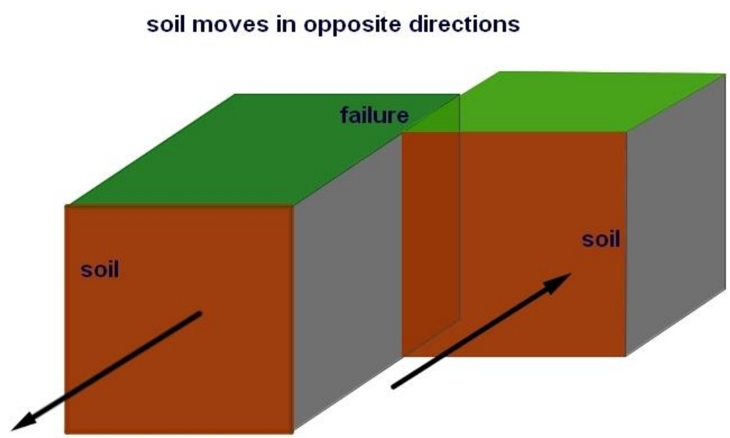
Soil failure (Glissement de terrain) , Québec
Maison des Préfontaines

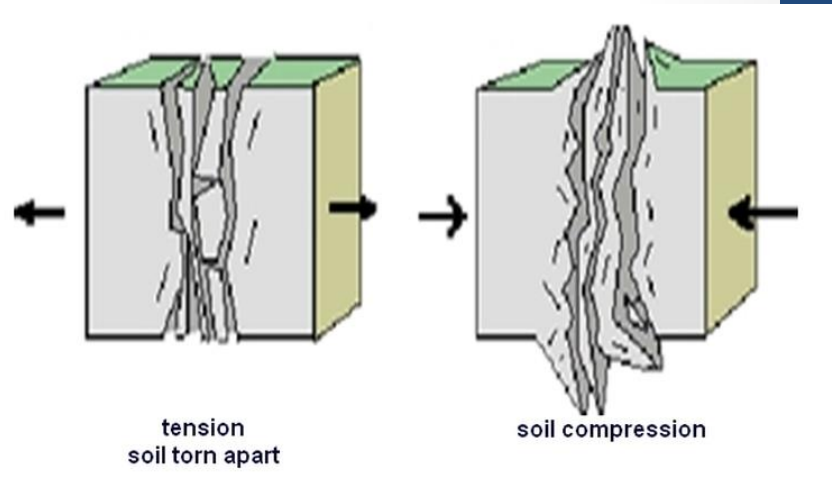
Soil failure (Landslide @ Lac du Chambon, 2015, Isère, France)

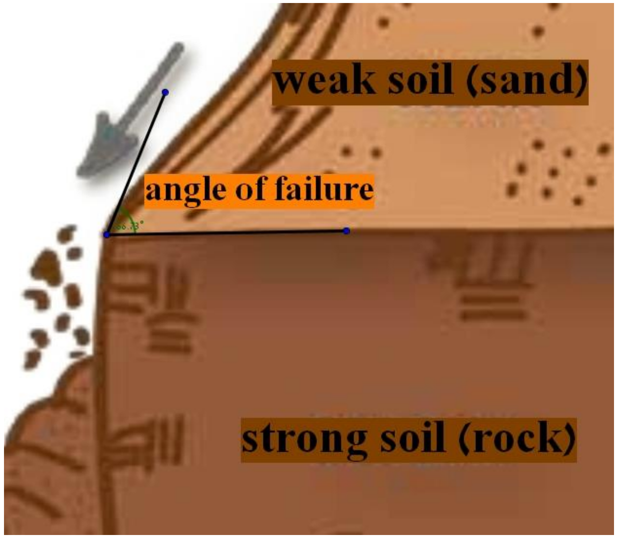
Water flows between sand and clay

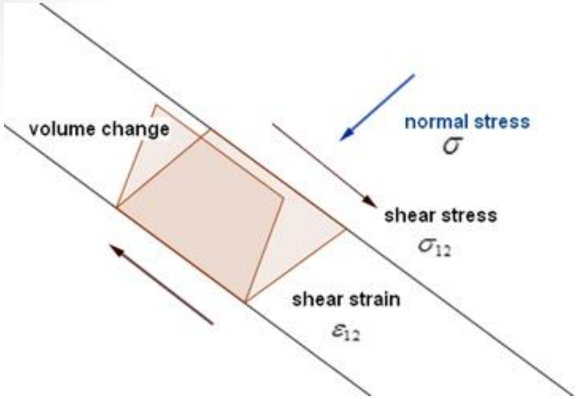
http://www.lapresse.ca/actualites/quebec-canada/201005/12/01-4279570-le-quebec-champion-des-glissements-de-terrain.php

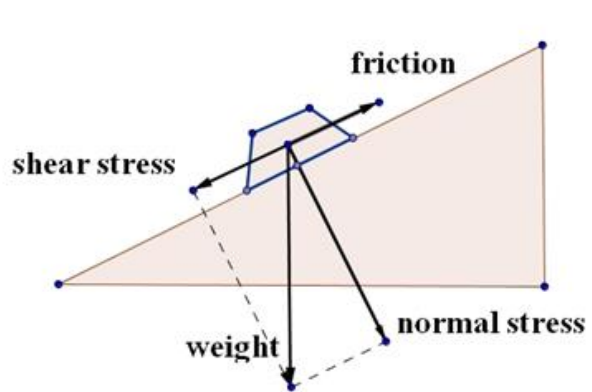
Concrete meaning
an application in soil mechanics











The transverse strain (strain due to the shear stress) $\varepsilon_{12}(\varphi)$ at the failure is connected to friction angle via the shear stress at failure, according to the following equation:

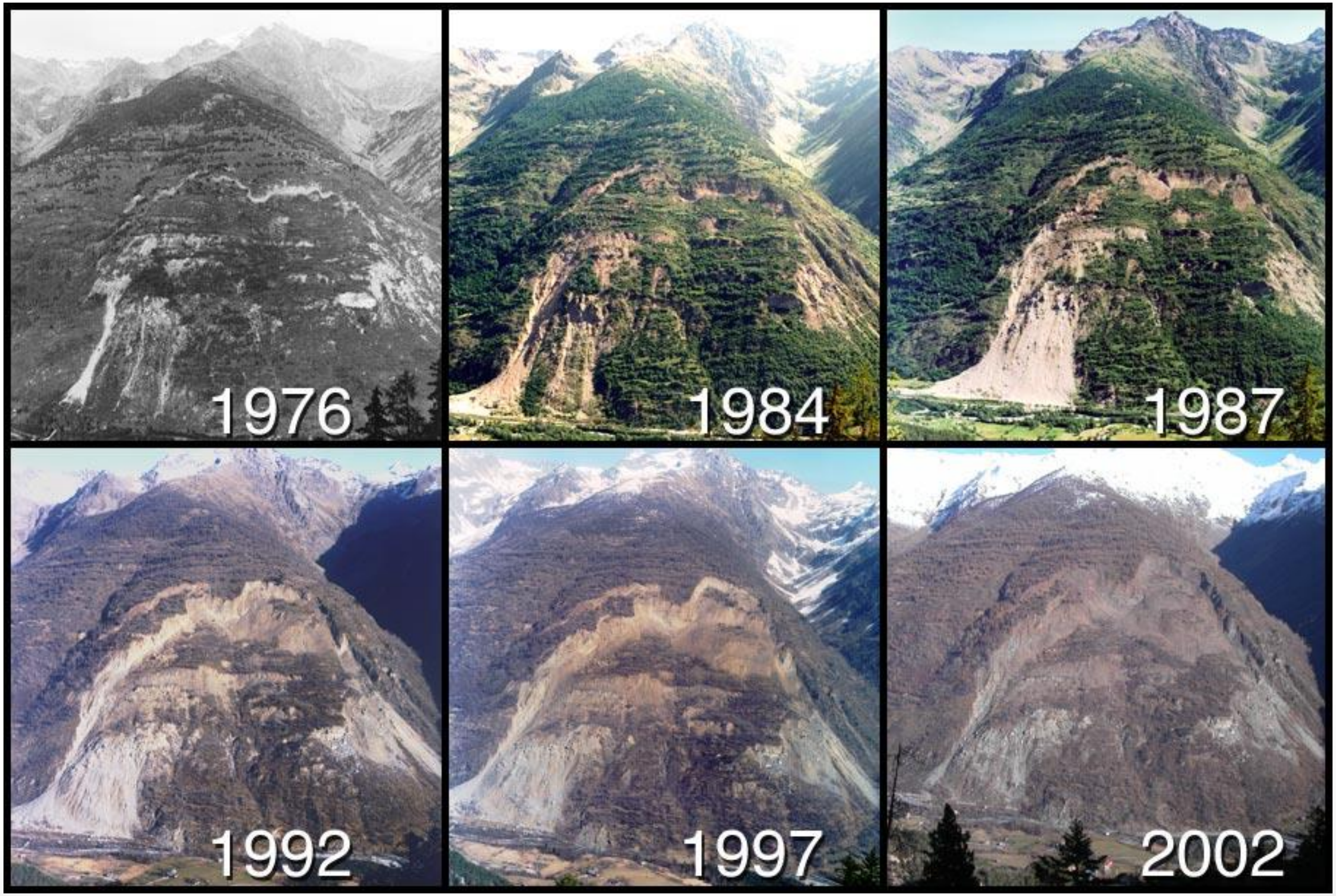
$$\varepsilon_{12}(\varphi) = A\cdot\left(\sigma_z \tan\varphi\right)^m$$

where σ_z is the constant weight of the soil, and fr clayey soil, at the failure the friction angle vary from 0 to $\pi/4$.

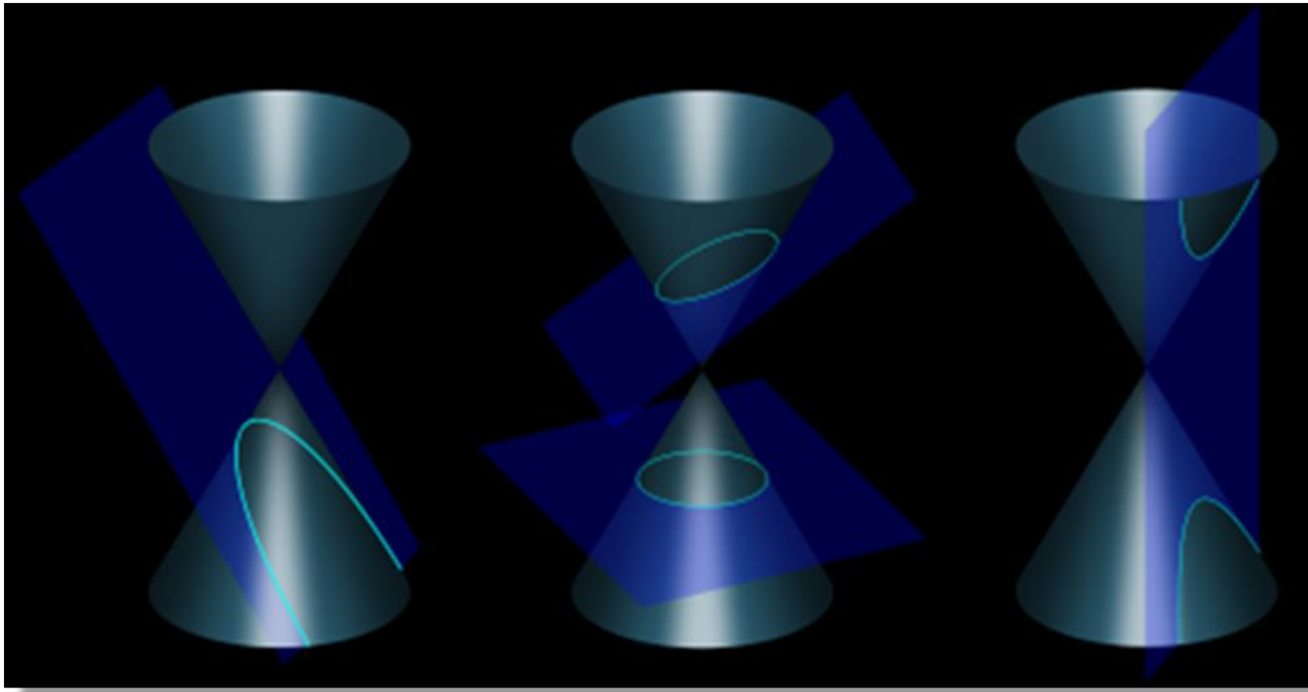
An expression for the total transversal deformation *E* :

$$E = \int_0^{\pi/4} \varepsilon_{12}(\varphi)\, d\varphi = A\sigma_z^m\, I_m$$

One of the biggest soil failures (landslide) in Europe: La Clapière, Saint-Étienne-de-Tinée, France

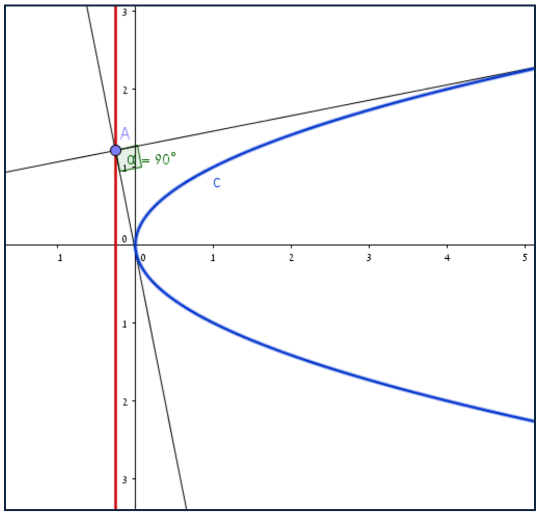


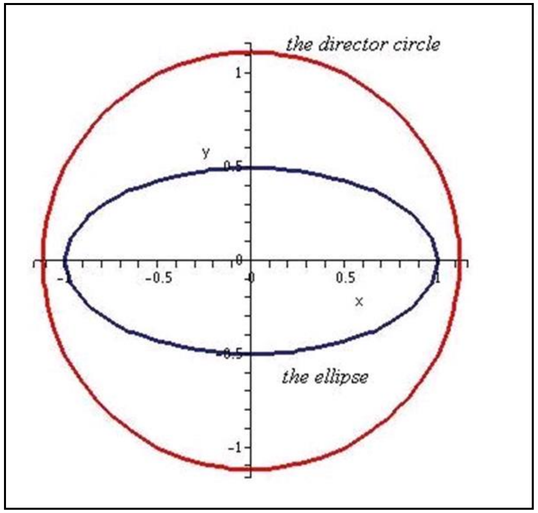
Joint work with Nurit Zehavi and Giora Mann (Weizmann Institute)

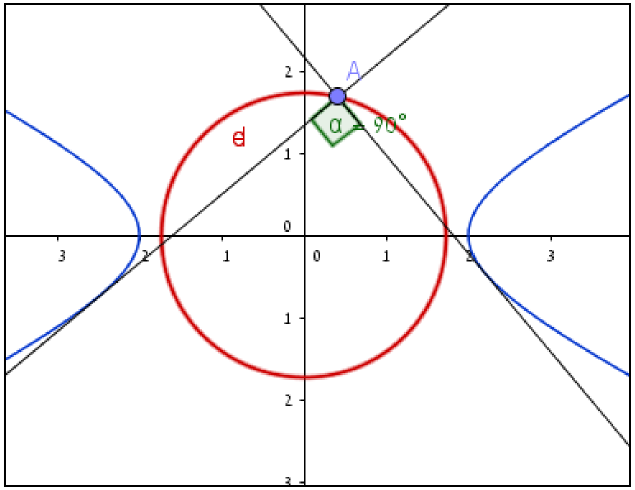


ANALYTIC GEOMETRY: BISOPTIC CURVES – FROM CONIC TO TORIC SECTIONS

Analytic geometry – directrix/director circle







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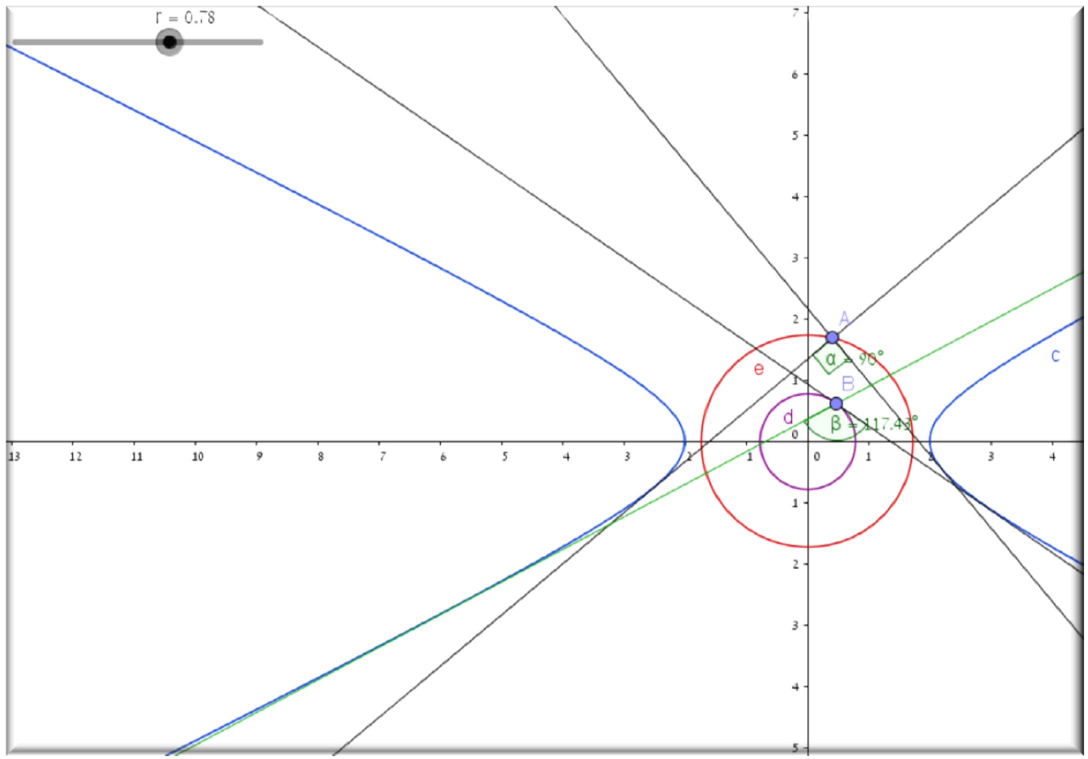
Analytic geometry: (b)isoptic curves



Examples of isoptic curves of an ellipse

We consider the ellipse whose equation is $x^2 + ky^2 = 1, k > 0$

We consider now the general case: none of the tangents in the pair is parallel to the y-axis, therefore both have a slope. Take a point $T(x_0, y_0)$; a line L through T and non-parallel to the y-axis has an equation of the form $y = m(x - x_0) + y_0$, where m is the slope of L. An ellipse has no singular point, thus Bezout's theorem (Berger 1996, section 16.4) ensures that the line L is tangent to the ellipse E if, and only if, it has a "double" point of intersection with the ellipse. The possible slopes are the following:

$$m_1 = \frac{\sqrt{x_0^2 + y_0^2 k^2 - 1} - kx_0 y_0}{k\left(1 - x_0^2\right)} \quad \text{and} \quad m_2 = -\frac{\sqrt{x_0^2 + y_0^2 k^2 - 1} + kx_0 y_0}{k\left(1 - x_0^2\right)},$$

where $x_0^2 + y_0^2 k^2 > 1$.

For tangents non parallel to the y-axis whose respective slopes are m_1 and m_2, the condition is equivalent to $\dfrac{m_1 - m_2}{1 + m_1 m_2} = \tan\theta$.

The requested geometric locus is determined by the equation

(13) $$\frac{2k\sqrt{k^2 y_0^2 + x_0^2 - 1}}{1 - k^2(x_0^2 + y_0^2 - 1)} = \tan\theta .$$

Denote $t = \tan\theta$ and square both sides of this equation. We obtain the following equation:

(14) $$\frac{4k^2(k^2 y_0^2 + x_0^2 + 1)}{\left(1 - k^2(x_0^2 + y_0^2 - 1)\right)^2} = t^2 .$$

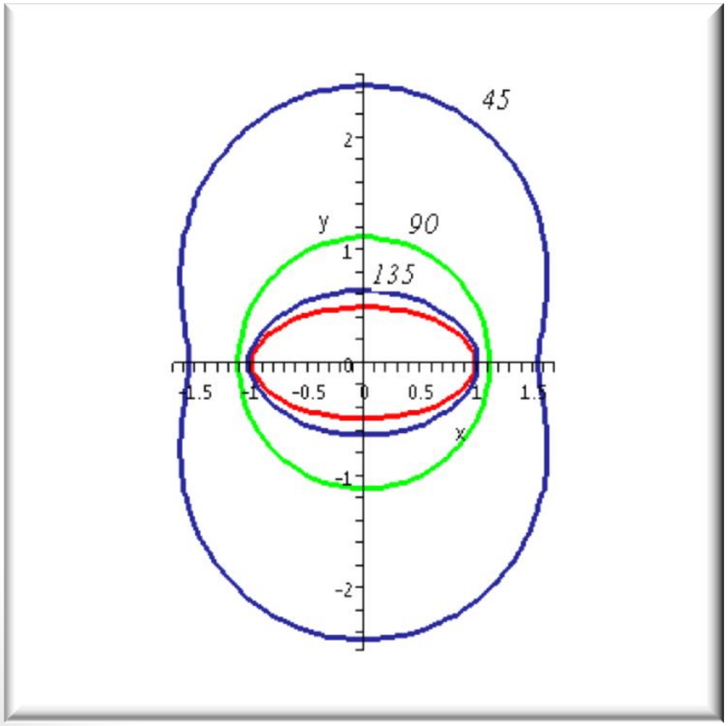
Squaring Whence doubling the solutions

Actually, the vanishing points of the denominator are points though which pass a suitable pair of tangents, one of the tangents being parallel to the y-axis. Multiplying both sides by the common denominator, we obtain the following polynomial equation of degree 4:

(15) $$k^4t^2x^4 + 2k^4t^2x^2y^2 - 2k^2(k^2t^2 + t^2 + 2)x^2 + k^4t^2y^4$$
$$- 2k^2(k^2t^2 + 2k^2 + t^2)y^2 + k^4t^2 + 2k^2(t^2 + 2) + t^2 = 0$$

The geometric locus of points from which the given conic ellipse E is viewed under a given angle θ is a called the ***θ-isoptic curve*** of the ellipse E for the given angle θ; we denote this curve by $OPT(k,\theta)$. The orthoptic curve of E is $OPT(k,90)$. In Figure 6, we show the curves $OPT(2,45)$, $OPT(2,135)$ and $OPT(2,90)$. The equation describing together the first one and the last one is $16x^4+32x^2y^2+16y^4-56x^2-104y^2+41=0$. This equation can be written under the form

(16) $$\left(x^2+y^2\right)^2-\frac{7}{2}x^2-\frac{13}{2}y^2+\frac{41}{16}=0.$$



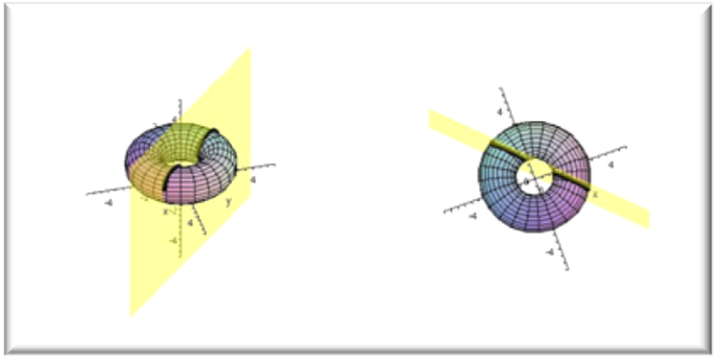
Quartic – spiric curves

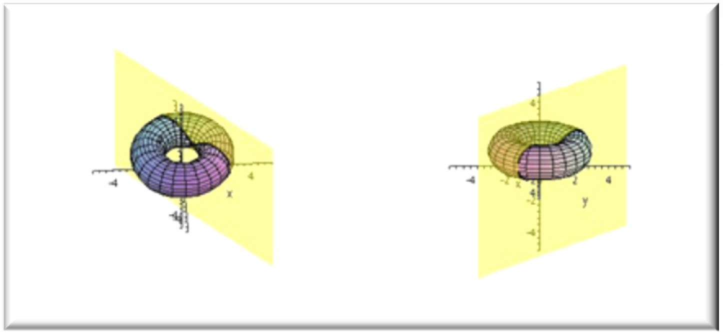
$$k^4t^2x^4 + 2k^4t^2x^2y^2 - 2k^2(k^2t^2 + t^2 + 2)x^2 + k^4t^2y^4$$
$$- 2k^2(k^2t^2 + 2k^2 + t^2)y^2 + k^4t^2 + 2k^2(t^2 + 2) + t^2 = 0$$

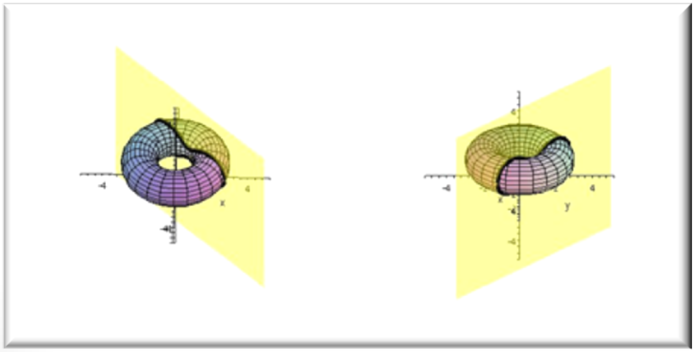
This equation determines a (specific) quartic called a **Spiric of Perseus.**

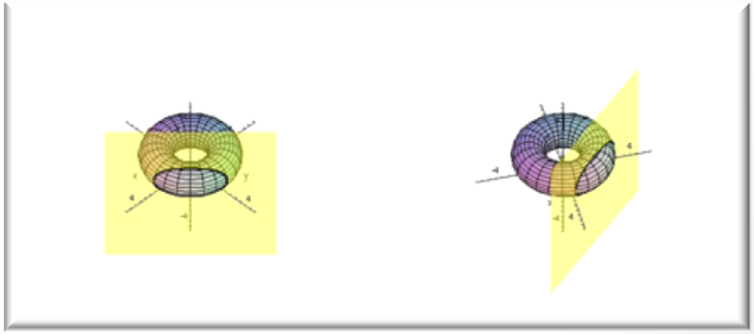
A spiric curve is the intersection of a torus of revolution with a plane parallel to the axis of the torus (Wassenaar, etc.)

Toric intersections with a plane parallel to the torus axis

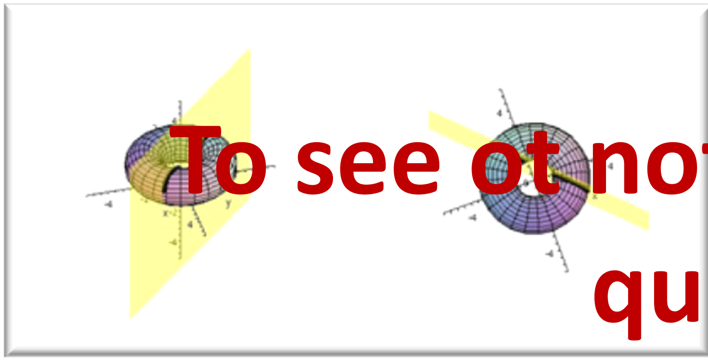






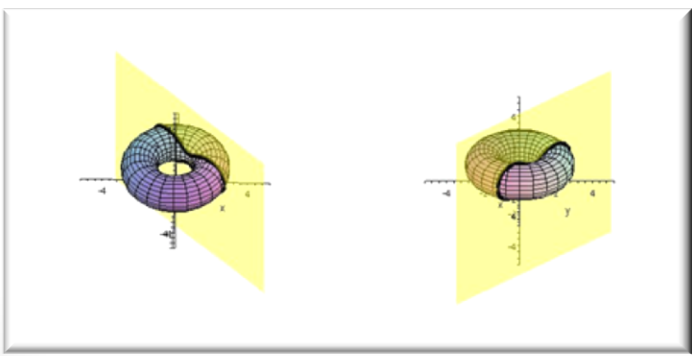


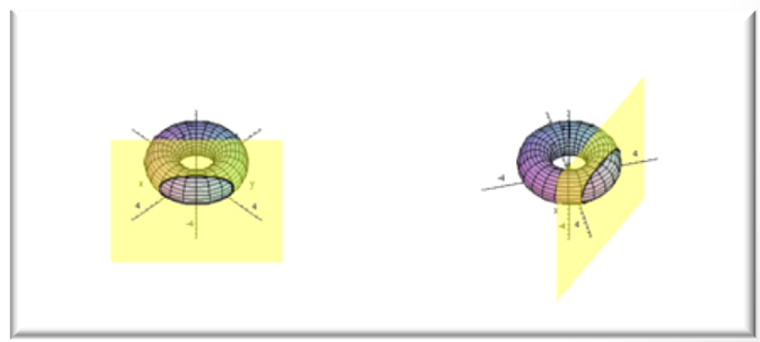
Toric intersections with a plane parallel to the torus axis



To see ot not to see, that’s the question!!!

(DP & Kidron IJTME 2006 - DP, Kidron and Zeitoun IJTME 2008)



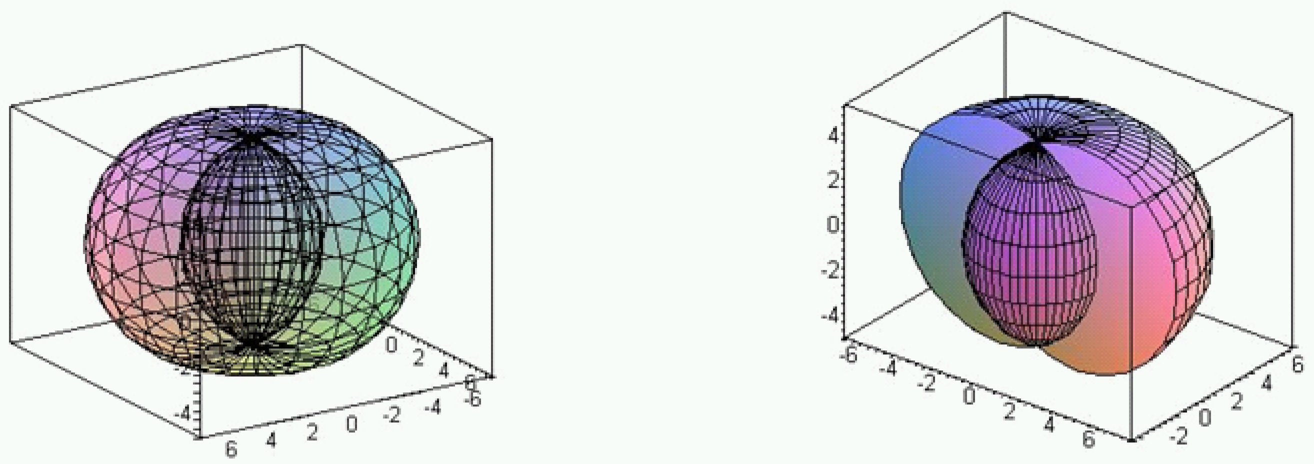


Two kinds of tori

(a) Non self intersecting
R=5, *r*=2

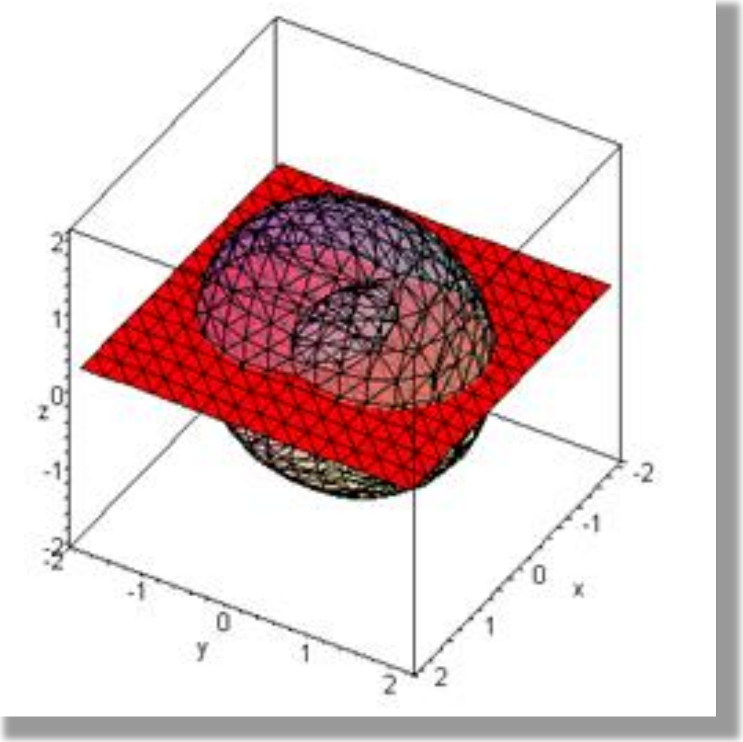
(b) self-intersecting
R=4, *r*=5

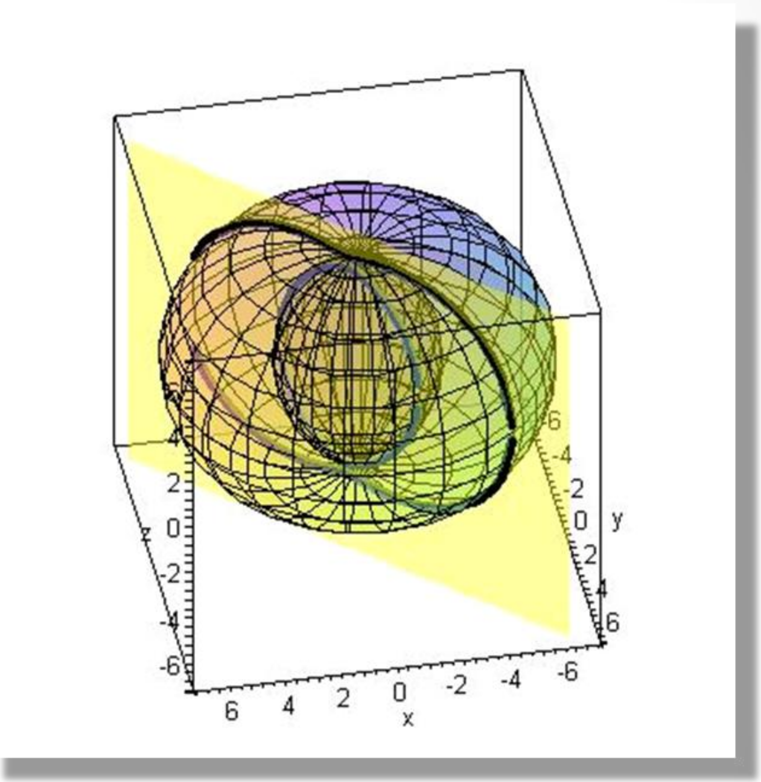
Self-intersecting torus



(a) The torus (b)View from inside

A toric intersection – spiric curve





D-P, Mann & Zehavi (2011): From conic intersections to toric intersections: the case of the isoptic curves of an ellipse, The Montana Mathematical Enthusiast 9 (1), http://www.math.umt.edu/TMME/vol9no1and2/index.html.
D-P, Zehavi & Mann (2014): Bisoptic curves of a hyperbola, International Journal of Mathematical Education in Science and Technology 45 (5), 762-781.

ENVELOPES OF 1-PARAMETER FAMILIES OF PLANE CURVES

René Thom: Sur la théorie des enveloppes. J. Math. Pures et Appliquées, XLI,fasc. 2, 1962, pp. 177-192, « manuscrit reçu le 25 avril 1960 ».

La récente réforme des études de licence en mathématiques a complètement évincé des programmes la théorie des enveloppes. … je ne puis que trouver cette disparition très regrettable; rappelons … le rôle des enveloppes dans la théorie des équations différentielles (intégrales singulières), et des équations aux dérivées partielles; mais est-il concevable qu'un professeur de lycée ait quelque usage des problèmes de la Géométrie Elementaire, sans connaître …. les phénomènes généraux de cette théorie?

…

Même d'un point de vue pratique, la théorie des envelopes rend compte de phénomènes familiers, sans elle inexpliqués; pour s'en convaincre, il suffit d'observer, à l'intérieur d'un bol hémisphérique de café au lait convenablement éclairé, la structure cuspidale des caustiques de réflexion, et leur variation quand l'éclairage se modifie.

Why envelopes disappeared from the curriculum

1. The classical theory is not rigorous enough
2. Too many particular cases: fixed points, singular points, stationary curves, etc.
3. Nothing ensures that all the “pathological” cases have been included in a catalogue
4. Actually: the theory is so rich that it is impossible to force it into the rules of a rigorous pedagogy.

René Thom: Sur la théorie des enveloppes. J. Math. Pures et Appliquées, XLI,fasc. 2, 1962, pp. 177-192, « manuscrit reçu le 25 avril 1960 ».

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Technology may help to address these various problems and revive the topic:

1. **Enquiry:**
   a. **Exploration with DGS and/or CAS**
   b. **Analytic work by hand/CAS**
2. **Algorithmization**
3. **Theoremization (Rasmussen, Wawro, & Zandieh, ESM 2015)**
4. **Possible byproducts:**
   a. **New mathematical knowledge: surfaces, singularities, applications, etc.)**
   b. **New computing skills: parametric solutions, algorithms based on Gröbner bases computations (Pech 2007, Buchberger's keynote at ACA 2015, DP-Zehavi iJMEST 2016)**

Why to revive these topics

- Interesting topic per se (not valid for all students)
- Nowadays provides a blended activity
- Envelopes may connect different topics

Sprinklers ->

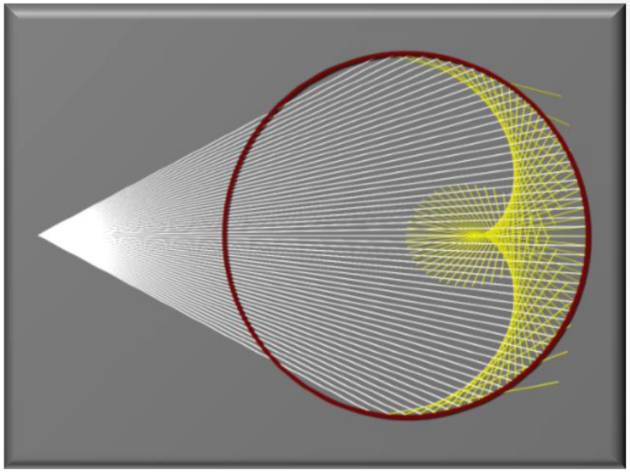
Pyrotechnical safety ->

Why to revive these topics

- Applications in science and engineering
  - Theory of Singularities
  - Geometrical Optics: Theory of Caustics, Wave Fronts

A **caustic** is the envelope of light rays reflected or refracted by a curved surface or object, or the projection of that envelope of rays on another surface. The caustic is a curve (or surface) to which each of the light rays is tangent, defining a boundary of an envelope of rays as a curve of concentrated light. Therefore in the image to the right, the caustics are the bright edges. These shapes often have cusp singularities.

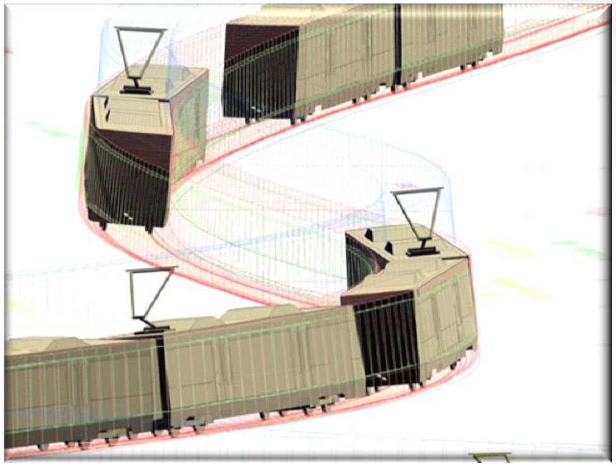
Ref: Arnold 1976, quoted by Capitanio 2002, and Thom 1962.

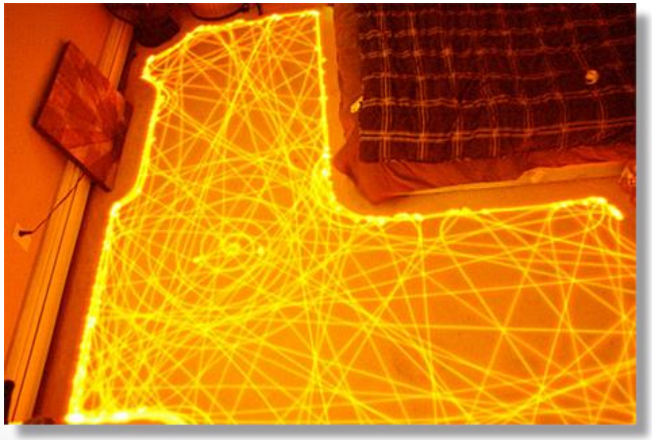


https://mathcination.wordpress.com/2016/02/15/how-algebra-sheds-light-on-things/

Why to revive these topics

- Applications in science and engineering
  - Theory of Singularities
  - Geometrical Optics: Theory of Caustics, Wave Fronts
- Robotics and kinematics: rigid body motion in the plane, in 3-space, collision avoidance of robot motion, construction of gears, etc. (Pottman and Peternell 2000).





Three descriptions of the envelope of a 1-parameter family of plane curves C_t

synthetic	**impredicative**	analytic

E is the union of the characteristic points; the characteristic point M_t is the limit point of the family of intersections $C_t \cap C_{t+h}$ as $h \to 0$.

Three descriptions of the envelope of a 1-parameter family of plane curves C_t

synthetic | **impredicative** | analytic

E is the union of the characteristic points; the characteristic point M_t is the limit point of the family of intersections $C_t \cap C_{t+h}$ as $h \to 0$.

Which space? Which topology?

The definition is not rigorous!!!

Three descriptions of the envelope of a 1-parameter family of surfaces S_t in 3-space

synthetic

E is the union of the characteristic points; the characteristic point M_t is the limit point of the family of intersections $C_t \cap C_{t+h}$ as $h \rightarrow 0$.

impredicative

E is a curve such that at each of its points, it is tangent to a unique curve from the given family. The locus of points where *E* touches C_t is called the *E*-characteristic M_t

analytic

Three descriptions of the envelope of a 1-parameter family of plane curves C_t

synthetic

E is the union of the characteristic points; the characteristic point M_t is the limit point of the family of intersections $C_t \cap C_{t+h}$ as $h \to 0$.

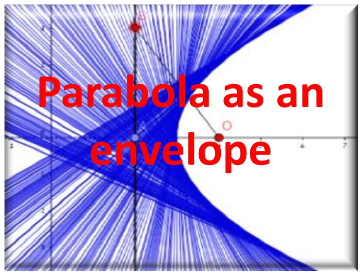
impredicative

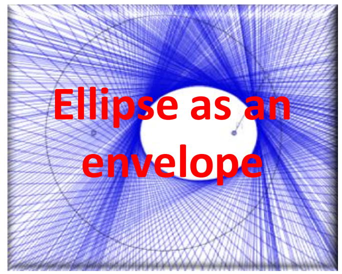
E is a curve such that at each of its points, it is tangent to a *unique* curve from the given family. The locus of points where *E* touches C_t is called the *E*-characteristic M_t

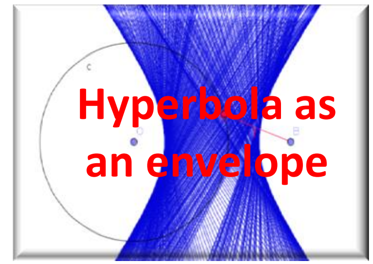
analytic

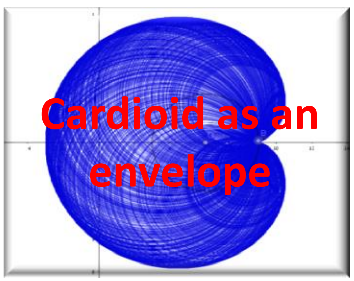
Uniqueness is not clear!!! No construction algorithm!!!!

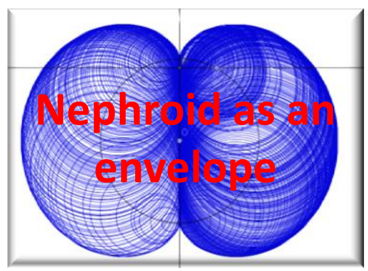
Here technology may help







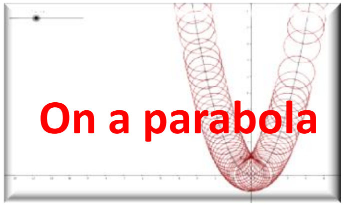


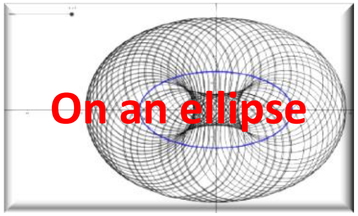


Experimental work using dynamical geometry

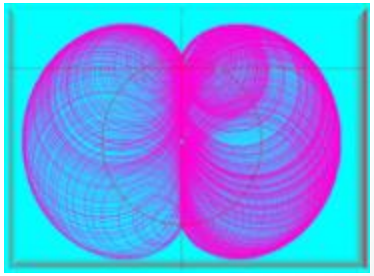
With a slider bar

- A family of circles centered





- Pros and cons:
    - Uses analytic presentation (equations)
    - More uniform repartition of the curves in the family
    - Not always useful: the example of a nephroid



-

TRANSITION FROM 2D TO 3D: ENVELOPES OF PARAMETRIC FAMILIES OF SURFACES

Three descriptions of the envelope of a 1-parameter family of surfaces M_t in 3-space

synthetic

E is the union of the characteristics; the characteristic C_t is the limit curve of the family of intersections $M_t \cap M_{t+h}$ as $h \to 0$.

impredicative

E is a surface such that at each of its points, it is tangent to a unique surface from the given family. The locus of points where *E* touches S_t is called the *E*-characteristic C_t

analytic

Two descriptions, according to whether the family is given by an implicit or a parametric presentation.

Analytic presentations of an envelope of a 1-parameter family of surfaces

implicit

The envelope of a 1-parameter family of surfaces given by an equation $F(x,y,z,c)=0$ is determined by the solution of the system of equations:

$$\begin{cases} F(x,y,z,c)=0 \\ \dfrac{\partial F(x,y,z,c)}{\partial c}=0 \end{cases}$$

For polynomial equations, use Gröbner bases algortihms

parametric

The envelope of a 1-parameter family of surfaces given by a parametrization $(M(u,v,t))_{u,v}$, is determined by the solution of

$$\det\left(\frac{\partial \overrightarrow{M}}{\partial u}, \frac{\partial \overrightarrow{M}}{\partial v}, \frac{\partial \overrightarrow{M}}{\partial t}\right)=0$$

Elimination of one of the three parameters $u,v,t,$ yields a parametrization of the envelope.

Transition form 2D to 3D

- The equations look similar
- The algebraic part of the work follows the same path
- Some of the graphical features of the software are not available anymore.
- This last point make the transition **critical**.

A 1-parameter family of planes

$$x + ty + t^2 z = t^3$$

$$t \in \mathbb{R}$$

We find:

1. A parametric presentation of the envelope
2. An implicit presentation of the envelope
3. A description of the cuspidal edge of the envelope

Problem with implicitization

Solving the system

$$\begin{cases} F(x,y,z,t)=0 \\ \dfrac{\partial F}{\partial t}(x,y,z,t)=0 \end{cases}$$

yields a parameterization of the envelope

$$\begin{cases} x = x(u,v) \\ y \doteq y(u,v) \\ z = z(u,v) \end{cases}$$

Question: when is it possible to find an implicit form for an equation of the envelope?

Answer: not always.

Partial solution: approximate implicitization (T. Schultz and B. Juttler 2010)

A 2-parameter family of planes

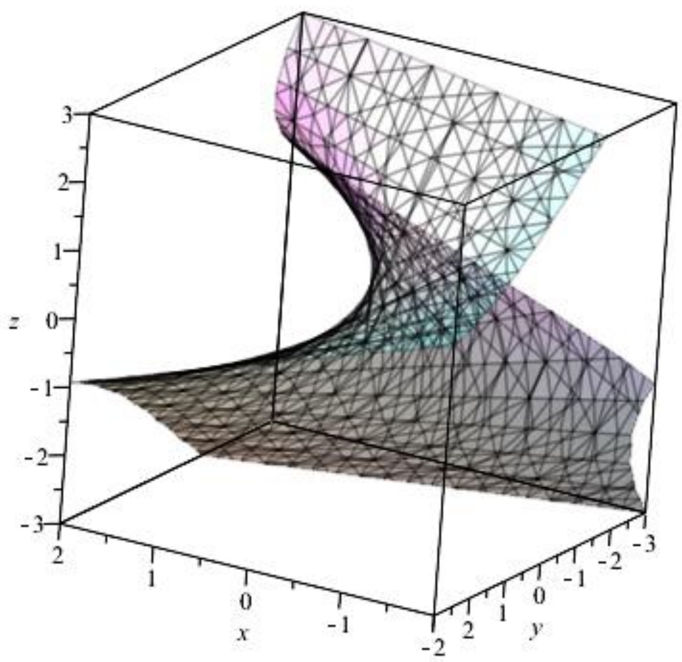
$$x+(u+v)y+(u^2+v^2)z=(u^3+v^3)$$
$$u,v\in R$$

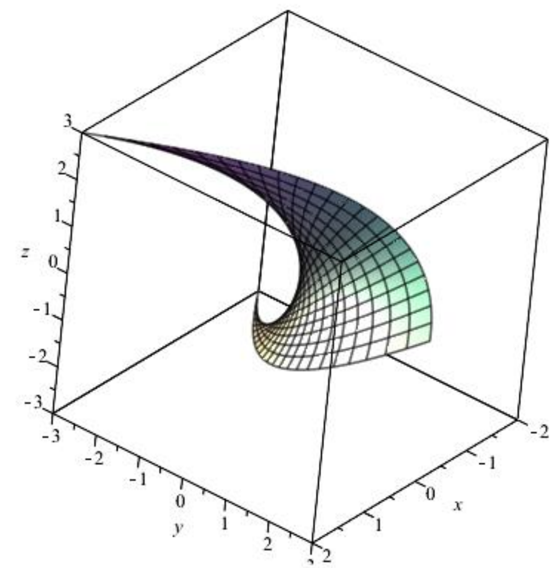
We look for:

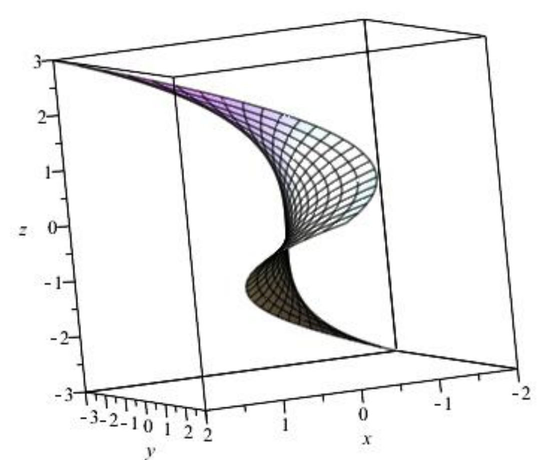
1. A parametric presentation of an envelope
2. An implicit presentation of this envelope

Wait a minute, we will have a surprise!!!!

Result comparison: first glance

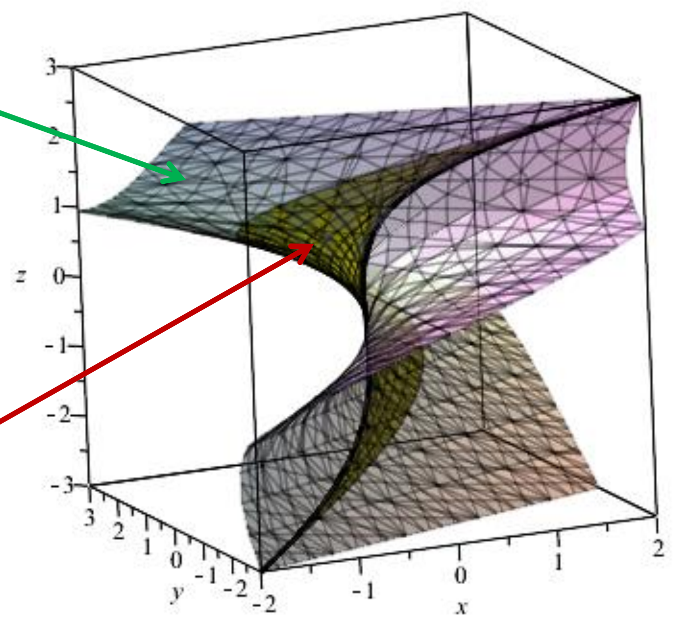






One step further

The surface determined by the implicit equation



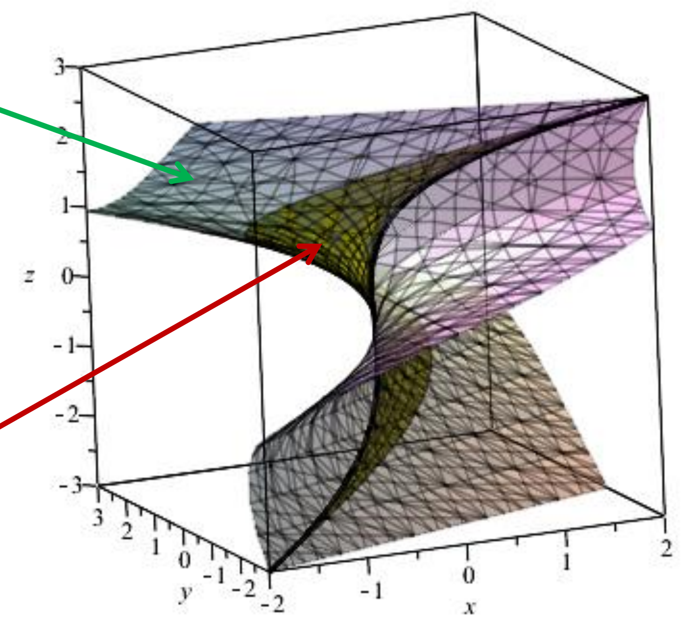
The surface determined by the parametric presentation

One step further

The surface determined by the implicit equation

Implicitization is one-way process, i.e. a logical implication, not a logical equivalence

The surface determined by the parametric presentation



The envelope is a variety with a boundary!!!!
It seems that we could not have discovered that without the technology

Revival? We may obtain more than that!!!!

- To pour new contents into a classical topic
- To have a new insight into a classical topic
- To bring an old-classical topic which has been « forgotten » back to the stage
- **To discover new results (which may not have been seen without technology), i.e.**
    - **Explore the student's Zone of Proximal Developement (Vigotsky, 1978).**
    - **Develop students' creativity**
- **The Gröbner bases methods may be used in higher dimensions, where the graphical methods are inexistent**

A PROJECT WHICH BEGAN RECENTLY
(WITH ZSOLT LAVICZA, KRISTOF FENYVESI, SARA HERSHKOVITZ)

The Tirgu Mures synagogue, built 1900

Not half a circle

Fibonacci numbers

Rundbogenstil

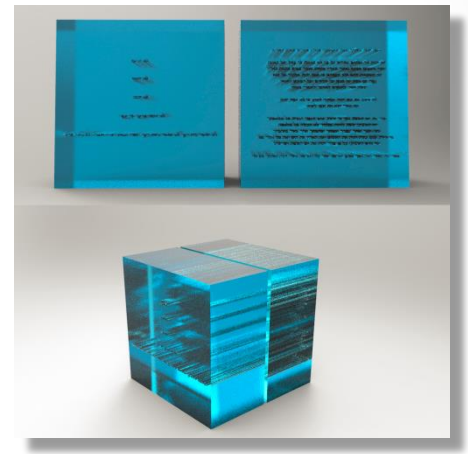
The Tirgu Mures synagogue, built 1900

The Wijnitzer Klaus synagogue in Sighet, built 1885

Symmetry in Tables of the Covenant

Synagogue in Lausanne

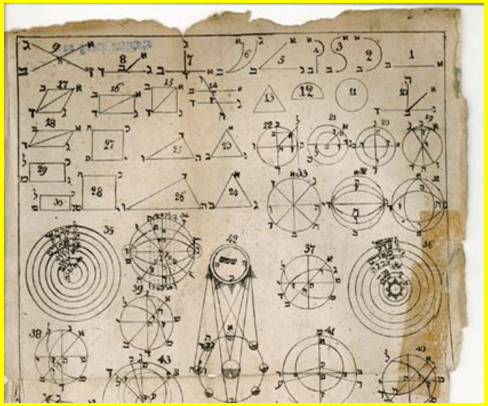
Prague: the
Spanish Synagogue



The issue of the tables' symmetry has been addessed in details by **Rabbi Moshe ben Yossef di Trani** (1500-1580), one of the most important Talmudists from his time until today, in Safed (Galilee).

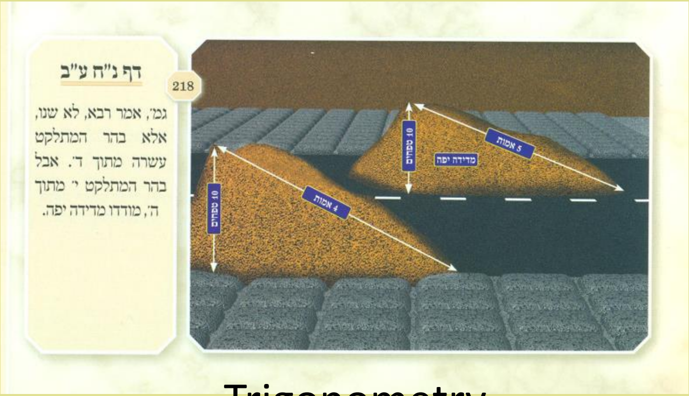
Source: DP, talk @ Symmetry Festival, Vienna 2016 and a submitted paper

Jewish Maths (Talmud)

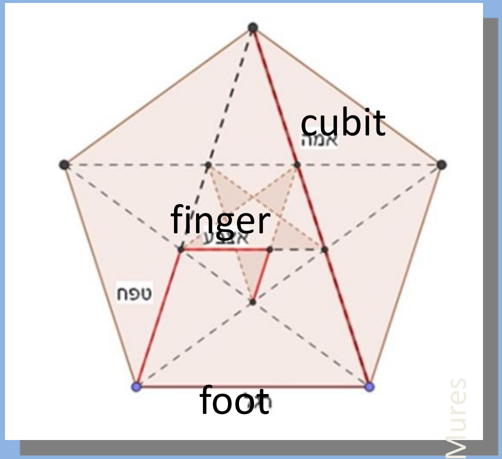


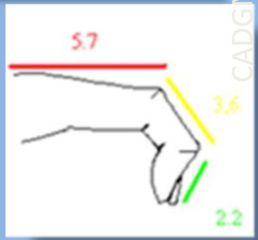
Computing the Jewish Calendar (lunar and solar)

An approach looking as integral calculus



Trigonometry





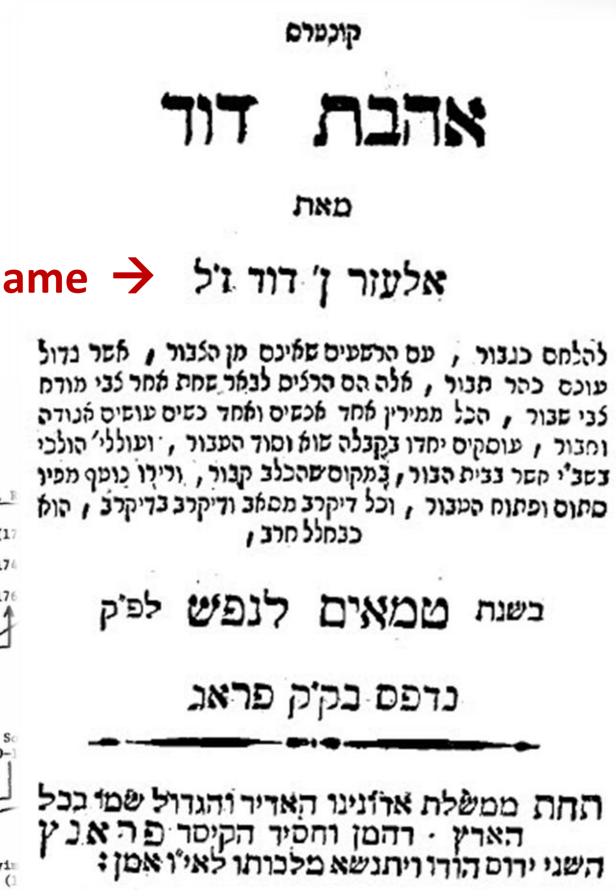
Biblical length units: Fibonacci numbers

Ref: DP (2014): JCRME2.

Rabbi-student relationship

Software by Y. Hacohen-Kerner, 2010

The author's name →


קונטרס

אהבת דוד

מאת

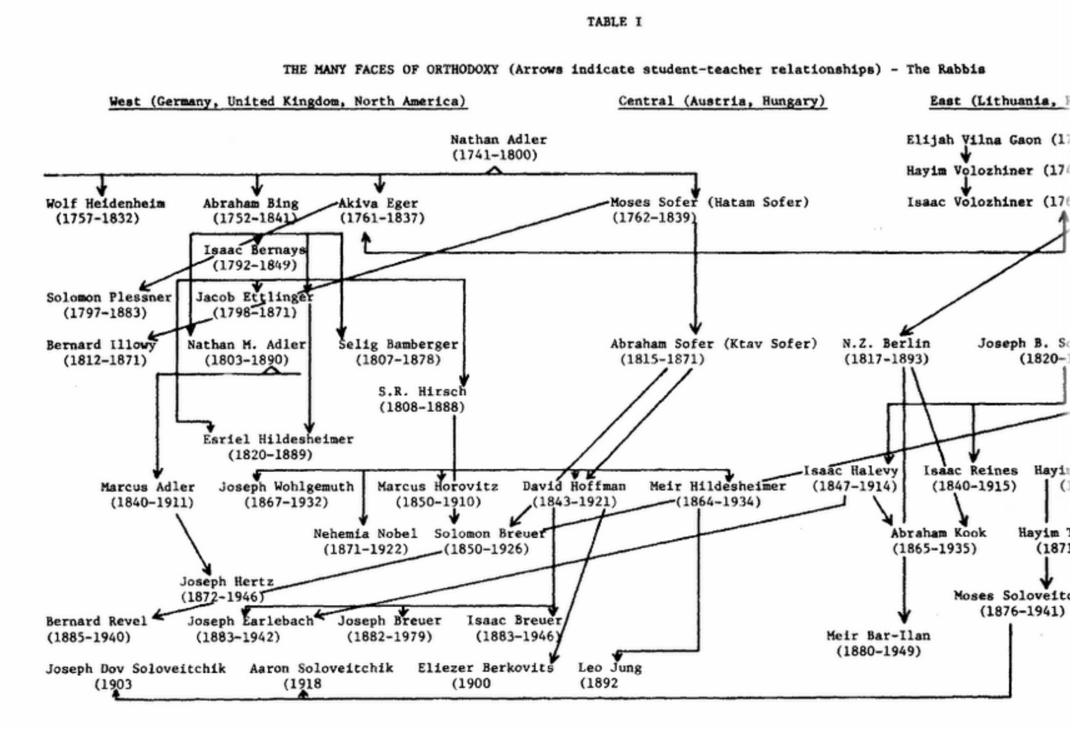
אלעזר ן' דוד ז"ל

להלחם כגבור , עם הרשעים שאינם מן הגבור , אשר גדול
עוכס כהר תבור , אלה הם הרלים לבאר שחת אחר לבי מורח
לבי שבור , הכל ממירין אחר אכשים ואחד כשים עושים אגודה
וחבור , עוסקים יחדו בקבלה שוא וסוד העבור , ועוללי' הולכי
בשבי' חסר בבית הבור , במקום שהכלב קבור , וירדו כנוטף מפיו
סתום ופתוח העבור , וכל דיקרב מסאב ודיקרב בדיקרב , הוא
כבחלל חרב ,

בשנת טמאים לנפש לפ"ק

נדפס בק"ק פראג

תחת ממשלת אדונינו האדיר והגדול שמו בכל
הארץ · רהמן וחסיד הקיסר פ ר א נ ץ
השני ירום הודו ויתנשא מלכותו לאי"ו אמן :



This gives the info on the period

CADGME 6 in Tirgu Mures

Revival: what do we mean?

- To pour new contents into a classical topic

Parametric integrals

- To have a new insight into a classical topic:

- **Architecture**
- **Art**

- To bring an old-classical topic which has been « forgotten » back to the stage

Differential Geometry:
- **Isoptic curves**
- **Envelopes**

All this is part of STEAM education

CONCLUSIONS?

ICT environment: CAS, DGS, databases, … the choices

- Button driven vs command driven: depends on the student/educator
- More than one CAS/DGS:
  - Switching from one register A of representation to another register B is not always available
  - Every CAS switches from algebraic representation to geometrical representation
  - Reverse switching, from geometry to algebra, is generally not given
- Dynamical features
  - Moving around with the mouse
  - For studying loci, a **Trace** feature
  - More important: **a slider bar**

ICT environment: CAS, DGS, databases, …

- Button driven vs command driven: depends on the student/educator
- More than one CAS/DGS:
  - Switching from one register A of representation to another register B is not always available
  - Every CAS switches from algebraic representation to geometrical representation

Danger: black box usage!!!!

  - GeoGebra switches automatically from geometry to algebra, others generally do not
- Dynamical features
  - Moving around with the mouse
  - For studying loci, a **Trace** feature
  - More important: **a slider bar**

Very important

- The technology:
  - Use more than one CAS/DGS
  - Web database

- Two basic functions of the usage of technology (Artigue 2002):
  - Pragmatic value
  - Epistemic value

- For the teacher:
  - The need for a « technological discourse » (Artigue 2002)

The influence of the institution

- Institutional culture:
  - Team work / multidisciplinarity
  - As a whole is there an institutional culture for STEAM education?
  - ICT enriched teaching, not only in maths
  - How ICTs are used in math education?
  - Which packages are available?
  - May new packages be purchased?

Al Cuoco and P.Goldenberg (1996)

The mathematics curriculum must be restructured to include activities that allow students to **experiment and build models** to help **explain** mathematical ideas and concepts.

Technology can be used most effectively to help students gather data, and test, modify, and reject or accept conjectures as they think about these mathematical concepts and experience mathematical research.

Conjecture, and then prove!!!

Artigue (IJCML - 2002)

What is firstly asked of software and computational tools is to be pedagogical instruments for the learning of mathematical knowledge and values which were defined in the past, mostly before these tools existed. The tools are also put forward to help in the fight against “inadequate” teaching practices: practices too much orientated towards pure lecturing or the procedural learning of mathematical skills

JERUSALEM COLLEGE OF TECHNOLOGY
LEV ACADEMIC CENTER

The Roland and Astrid Dana-Picard Chair for Mathematics, Education and Judaism

thank you
mulțumesc
köszönöm
Merci
hvala
danke
děkuji
obrigado
gracias
спасибі